Work for Digital Signal Processing Supervision I

Please attempt all the questions below and submit your work by 6pm on the day before the supervision. Regardless of whether you submit the main body of the work in PDF or hand-written form, please include in it any code samples and their output, and also send email them to me as separate files so that I can run/unit test them. Please also include all your working and any additional thoughts with your work.

Questions

1. What is the magnitude ratio between two signals if the power difference is
   (a) -6dB
   (b) 3dB
   (c) 20dB
2. What is the absolute power of a 3dBm signal? What is the absolute power of a -10dBW signal?
3. Examples sheet exercises 1, 2 and 4.
4. Pick any two of examples sheet exercises 3, 5, 6 and 7.
5. Design a digital filter to convert a 1kHz square wave into a perfect sine wave. The sampling frequency is 8kHz, and you may assume that the input has been suitably low-pass filtered prior to sampling. (*Hint*: Noting that the square wave is made up of only odd harmonics may help you to choose your cutoff frequency.) Test your design in MATLAB.
6. (a) Give three distinct reasons why one might "zero pad" an input sequence before applying the DFT, and explain why zero padding is appropriate in each of the three situations.
   (b) Why might one multiply the entire input sequence with a "window function" before attempting to estimate the spectral content of a signal? Aid your answer by constructing a suitable graphical example in MATLAB. Why might a `sinc` function at first appear to be an excellent window function? Why is it not used directly as a window function in practice?
7. (a) Describe the complete spectrum of a sampled signal. How can a sampled signal (sampled in such a way that it obeys the Nyquist sampling criterion) be exactly reconstructed? Explain how this operation is achieved by a reconstruction filter.
   (b) Show that a real signal of bandwidth B centred at frequency $\pm f_c$ can be exactly reconstructed provided that the sampling frequency f_s obeys
   $$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$
   where $m \in \mathbb{N}$ is an arbitrary integer.
   You may wish to use diagrams in your sketch proof.
   Show (informally) in the case $m = 0$ that this criterion reduces to the familiar law of sampling at a frequency of at least twice the highest frequency present in the input.
   (c) Explain the advantages, and principal disadvantage, of bandpass sampling ($m \neq 0$), compared with the familiar $m = 0$ case.
8. Examination question: 2012 Paper 8 Question 6 (attached at the end of this document).

6 Digital Signal Processing (MGK)

BBC Radio Cambridgeshire broadcasts a radio signal on a carrier frequency of 1026 kHz with a (double-sided) bandwidth of 10 kHz. You connect a long wire (antenna) via an amplifier and bandpass filter (0.5–2.0 MHz) to an analog-to-digital converter (ADC) with a sampling frequency of 5 MHz.

(*a*) You record $n = 500$ consecutive samples $(x_0, x_1, \dots, x_{499})$ from the analog-to-digital converter output and calculate the Discrete Fourier Transform (DFT)

$$X_k = \sum_{i=0}^{n-1} x_i \cdot \mathrm{e}^{-2\pi \mathrm{j} \frac{ik}{n}}$$

(*i*) For which index value(s) k do you expect $|X_k|$ to best indicate the received signal strength of this radio station? [4 marks]

(*ii*) What preprocessing step would improve this indication? [4 marks]

(*iii*) What redundancy do you expect to find in the DFT output vector X, considering that the input signal is real-valued? [2 marks]

(*iv*) Explain a technique that exploits this redundancy to calculate this real-valued DFT more efficiently. (You can assume that an FFT implementation is already available and that $n = 512$ is used instead.) [4 marks]

(*b*) You want to record the output of this radio station for later analysis, but you do not yet know how it was modulated. How can you convert the ADC output sequence $\{x_i\}$ such that the resulting sequence encodes efficiently what is happening in the frequency range 1021–1031 kHz, with as low a sample rate as possible? [4 marks]

(*c*) You finally learn that the signal recorded in (*b*) was an amplitude-modulated positive mono audio signal. How can you demodulate it? [2 marks]